WHOLE OF GCSE MATHS

No Waffle GCSE aim: To provide free top-notch education so that EVERYBODY can achieve the top grades

~ Maria Adams - No Waffle GCSE😃~

contents list:

Indices

Rules:

1. $x^{-n} = \frac{1}{x^n}$
2. $x^{\frac{1}{n}} = \sqrt[n]{x}$
3. $x^{-\frac{1}{n}} = \frac{1}{\sqrt[n]{x}}$
4. $x^{\frac{n}{m}} = (\sqrt[m]{x})^n$
5. $\sqrt{\frac{m}{n}} = \frac{\sqrt{m}}{\sqrt{n}}$
6. $\sqrt{a} \times \sqrt{a} = a$
7. $\sqrt{a} \times \sqrt{a} \times \sqrt{a} = a\sqrt{a}$
8. $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
9. $\left(\frac{x}{y}\right)^{-n} = \frac{y^n}{x^n}$
10. $\frac{x^{-n}}{y} = \frac{1}{y \times x^n}$

Powers of 10 addition + subtraction:

1. $(1.52 \times 10^5) + (5.4 \times 10^4)$

$(15.2 \times 10^4) + (5.4 \times 10^4) = 20.6 \times 10^4 = \underline{2.06 \times 10^5}$

2. $(7.29 \times 10^{15}) \div (9 \times 10^{-5})$

$\left(\frac{7.29}{10}\right)^{15} \div \left(\frac{9}{10}\right)^{-5} = \left(\frac{0.81}{10}\right)^{20} = 0.81 \times 10^{20} = \underline{8.1 \times 10^{19}}$

$15 - -5 = 20$

Rationalising the denominator

$\frac{6}{2+\sqrt{5}} \times \frac{(2-\sqrt{5})}{(2-\sqrt{5})} \quad \frac{12 - 6\sqrt{5}}{-1} = -12 + 6\sqrt{5}$

$(2+\sqrt{5})(2-\sqrt{5})$

$4 - 2\sqrt{5} + 2\sqrt{5} - 5 = -1$

$\boxed{\text{Anything} \div -1 = \times -1}$

Conversions

1. (a) convert 5 Km/h to m/s

$\frac{5 \times 1\text{Km}}{H}$

$\frac{5 \times 1000\,m}{3600} = 1.39\,m/s$

(b) 3m/s to km/h

$\frac{3 \times 1m}{s} \quad \frac{3 \times 0.001}{(1 \div 3600)} = 10.8\,\text{Km/h}$

$\boxed{\begin{array}{l}\text{Difference of 2 squares} \\ (a+b)(a-b) = a^2 - b^2\end{array}}$

$\boxed{\begin{array}{l}\text{LCM formula} \\ \text{LCM} = \text{HCF} \times \text{rest of no.}\end{array}}$

finding average speed

$$\text{Average Speed} = \frac{\text{total distance}}{\text{total time}}$$

Distance = 80 km   Distance = 64 km
Speed = 50 km/h   Speed = 80 km/h
Find the average speed :

1. find time

$t = 1.6h$ $t = 0.8h$

2. equation

$$\frac{80km + 64km}{1.6 + 0.8} = 60 km/h$$

expanding + factorising quadratics

a:)
~~fact~~ Solve $a^2 + 8a + 12$

$(a+2)(a+6)$
$a = -2$ or $a = -6$

b:)
$5x^2 + 16x + 3$

15 16
× +

$5x^2 + 15x + 1x + 3$
$5x(x+3)\ 1(x+3)$
$(5x+1)(x+3)$

complex factorising

$2(y+2)^2 + 4(y+2)$

$2(y+2)\left((y+2)+2\right)$
$2(y+2)(y+4)$

$(y+2)(y+2)$
$2(y^2+4y+4) + 4(y+2)$
$2y^2+8y+8 + 4y+8$
$2y^2+12y+16$

32 12
× +
$2(y^2+6y+8)$

$2y^2+4y+8y+16$

$2(y^2+2y$ $2(y+4)(y+2)$

Triangular numbers

1	3	6	10	15
1	1+2	1+2+3	1+2+3+4	1+2+3+4+5

Sequences

Deductive - Position to term rule (n^{th} term)
Inductive - term to term rule

a = first term
d = common diff
r = common multiplier

① Arithmetic
deductive $U_n = a + (n-1)d$
e.g 7, 10, 13, 16
$U_n = 7 + 3(n-1) = 4 + 3n$

Inductive $U_{n+1} = U_n + d$
e.g 4, 9, 14, 19
$U_{n+1} = U_n + 5$

② Geometric
deductive $U_n = ar^{n-1}$
e.g 3, 6, 12, 24
$U_n = 3 \times 2^{n-1}$
inductive $U_{n+1} = U_n \times r$
e.g 3, 6, 12, 24
$U_{n+1} = U_n \times 2$

Q → The 1st term of a geometric sequence is 4 and r = 5. Work out 10th term.

$a = 4$
$r = 5$

$U_n = ar^{n-1}$
$U_n = 4 \times 5^{10-1}$
$U_n = 4 \times 5^9$
$U_{10} = 7812500$

ar^{n-1} ~~4×~~ $3 \times 8^{n-1}$

RANGE is NOT an AVERAGE, it is a SPREAD OF DATA.

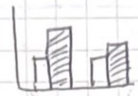
Bar Charts & Histograms

types of bar charts:

1. Simple

2. Compound

3. Comparative



Discrete → grouped data

Continuos → measurable data

- **Comparing data sets:**

(1) 1 measure of Central tendency (mean, mode, meadian)

(2) 1 measure of Spread (range or IQ range) → bigger range, less consistent

Bar Charts	Histograms
• Equal gaps	• no gaps
• Y-axis frequency	• Y-axis frequency
• X-axis grouped data	• X-axis continuos data

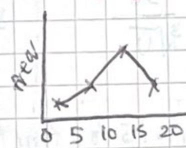
MOVING AVERAGES smooths out peaks and troughs cyclically

Frequency graphs

- frequency table

A	tallys	F
5-10	𝍸	5
11-15	|||	3
16-20	||||	4

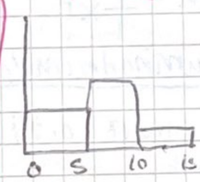
- frequency polygon



- grouped freq table

Height	Freq	mead
$140 \le x < 150$	50	145

- freq diagram



HOW TO FIND:

(a) mean
- Find MP, ~~MP×Freq~~
- Mid Point × Freq
- Divide total of MP×F by total F

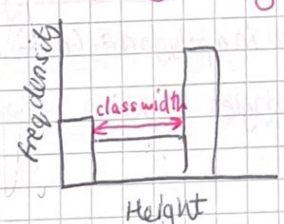
(b) Range

Find Highest − lowest on 1st Column

(c) Meadian

use frequency column

- **Frequency Density** = frequency × Class width → Area of Bar / Shape



Estimating Meadian from a histogram

Speed	freq
$0 \le v < 40$	12
$40 \le v < 60$	12
$60 \le v < 80$	14
$80 \le v < 100$	2
	40

① 40 (total) ÷ 2 +1 = 20

② Find which group 20 is in = $40 \le v < 60$

③ You need 20^{th} item of data which is the 8^{th} one. (12 + 8 = 20)

④ $\frac{8}{12}$ → item / → freq of that group × (prev + 8) + beg of class width

$\frac{8}{12} \times (12+8) + 40 = \underline{53.3}$ ✓

$\frac{8}{12} \times (\text{prev} + 8) + \text{beg of class width}$

Scatter graphs

- time goes on the x-axis
- Scattergraphs have bivariate data which is called correlation

Interpolation → using LOBF for a point

Extrapolation → extending LOBF

Reciprocals

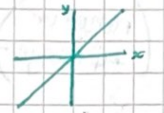
The reciprocal of a number (n) is $1 \div n$ AKA n^{-1}! ② if you $\times$ a num by its reciprocal you get 1.

Ratio & Proportion

Ratio compares part to part but fractions compare part to whole.

- Direct Proportion $[y \propto x \text{ OR } y = kx]$

K = Constant of proportionality



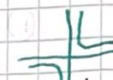
EX: M is directly proportional to p.
when m=8, p=18
work out the value of p
when m=13.

$M \propto P$
$M = K \times P$
$8 = K \times 18$
$K = \frac{8}{18} = \frac{4}{9}$

$m = \frac{4}{9} \times P$

$13 = \frac{4}{9} \times P$
$13 \div \frac{4}{9} = P$
$P = 29.25$

- Inverse proportion $[y \propto \frac{1}{x}, y = \frac{K}{x}$



EX: If it is known that H is inversely proportional to the cube root of v and H=12 and V=27
Find the value of v when H=18.

$y = \frac{K}{x} \longrightarrow H = \frac{K}{\sqrt[3]{v}}$

$12 = \frac{K}{\sqrt[3]{27}} = \frac{K}{3}$
$K = 36$
$H = \frac{36}{\sqrt[3]{v}}$

$18 = \frac{36}{\sqrt[3]{v}}$ $18 \times \sqrt[3]{v} = 36$
$v = 2^3 = 8$

Recurring decimals to fractions

a.) convert $0.\dot{2}\dot{1}$ to a fraction

$x = 0.2121212121...$
$100x = 21.21212121$
$100x - x = 21$
$99x = 21$
$x = \frac{21}{99} = \frac{7}{33}$

b.) Convert $0.1\dot{5}$ to a fraction

$x = 0.1555555..$
$10x = 1.5555555$
~~$100x - 1x =$~~
$100x = 15.555555$
$100x - 10x = 14$
$90x = 14$
$x = \frac{14}{90} = \frac{7}{45}$

Similar sollids

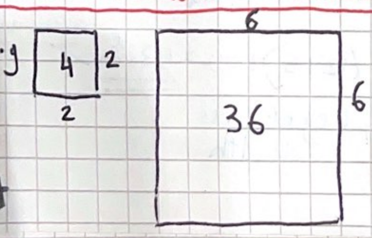
Amy says that $2.5cm^3$ is the same as $2.5mm^3$.
Is she right?

$1cm = 10mm$
$1cm^3 = 10mm^3$
↓
$1cm^3 = 1000mm^3$

Not equivalent, therefore Amy is incorrect.

$$\boxed{\begin{array}{l}\text{linear Scale Factor}^2 = \text{Area Scale Factor} \\ \text{linear Scale Factor}^3 = \text{volume scale Factor}\end{array}}$$

e.g



Side lengths $2 \overset{\times 3}{:} 6$ LSF = 3

$LSF^2 = ASF$ $3^2 = 9$

Area $4 \overset{\times 9}{:} 36$

Interior + Exterior angles

- Sum of interior angles in a polygon = $(n-2) \times 180°$
- Interior + Exterior angles = 180°

Using
Proving the q

EX1:

$x = -b$

If ax^2

x^2

$(x+$

$(x+$

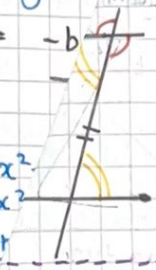
TRIGONOMETRY

Vertically opposite X

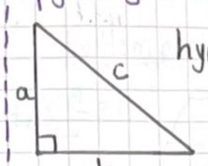
Corresponding F

Alternate Z

Cointerior
($A + B = 180°$)



Pythagoras' thereom



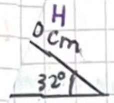
hypotenuse = longest side (c)

$a^2 + b^2 = c^2$

help to find missing side

S	O	H	C	A	H	T	O	A

finding missing lengths



Find x to 2dp.

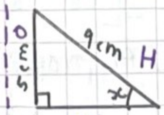
S O H

$\sin(32°) = \frac{x}{10}$

$\sin(32°) \times 10 = x$

$x = 5.30$ cm (2dp)

finding missing angles



Calculate x to 1dp.

$\sin(x) = \frac{5}{9}$ $\quad \sin^{-1}\left(\frac{5}{9}\right) = x$

inverse trignometric function

$= 33.7°$ (1dp)

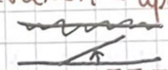
exact trignometric values

	0	30	45	60	90
Sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
Tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	X

Angles of depression + elevation

∡ depression = downwards from horizontal

∡ elevation = upwards from horizontal



GRAPHS

straight line graphs

linear formula → $y = mx + c$ (m → gradient, c → y-intercept)

y intercept → $x = 0$

x intercept → $y = 0$

gradient formula → m = vertical height (difference between y-coordinates) / horizontal distance (difference between x coordinates)

Parallel → m is same

Perpendicular → m × to get −1
so
$m = -\frac{1}{m}$

EX: Find the co-ordinates where the 2 lines intersect

$y = -x + 2$ and $y = (5 - 3x)/2$

$-x + 2 = (5 - 3x) \div 2$

$-2x + 4 = 5 - 3x$

$x = 1$

$y = -1 + 2$

$y = 1$

(1, 1)

EX2: Find the equation of the line that passes through (3, 4) and (5, 8)

$m = \frac{\text{y's difference}}{\text{x's differences}} = \frac{4}{2} = 2$

$y = mx + 2$ (plug in co-ordinates)

~~$4 = (m \times 3) + 2$~~

~~$2 = m \times 3$~~

$8 = 5 \times m + 2$

$6 = 5 \times m$

$m = \frac{5}{6}$

$y = \frac{5}{6}x + 2$

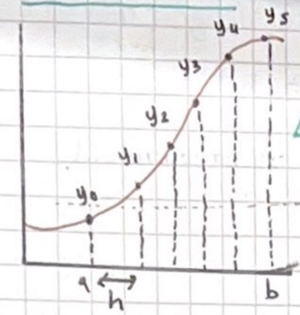
EX3: find the midpoint of the line segment (1, −2) and (7, 5)

midpoint $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$m = \left(\frac{8}{2}, \frac{3}{2}\right) = (4, 1.5)$

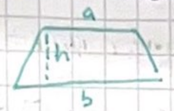
Velocity time graphs

- Area under curves



trapezium width
(AKA strip width)

long method:



$A = \frac{h}{2}(a+b)$

e.g. $\frac{h}{2}(y_0+y_1) + \frac{h}{2}(y_1+y_2) \dots$

short method

Notice → Start + End appear once, middle values repeat twice

e.g. $\frac{h}{2}\left[y_0 + y_5 + 2(y_1+y_2+y_3+y_4)\right]$

IN GENERAL, trapezium rule:

Area $\approx \frac{h}{2}\left[y_0 + y_n + 2(\underbrace{\text{middle values}}_{y_1+\dots+y_{n-1}})\right]$

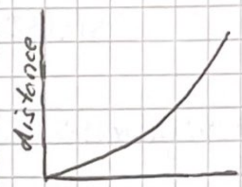
$\boxed{\text{acceleration} = \text{velocity} \div \text{time}}$
(m/s²) (m/s) (s)

- Features of a velocity time graph

Distance - area underneath

Gradient - acceleration

Features of a distance time graph



speed = gradient

velocity = rate + direction

Equations of m…

- If an object is travelling at c…

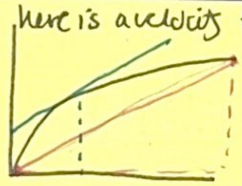
D
S T

Speed = d…

- If you're travelling at co…

S displacement
U initial velocity
V final / current velo…
A acceleration
T time

here is a velocity time graph of a car during 50 seconds.



(a) Work out the average acceleration during the 50 seconds
find gradient of straight line

(b) Estimate during the 50 seconds the time where the instantaneous acceleration = the average acceleration. tangent, point where has it touch

① $v^2 = u^2 + 2as$

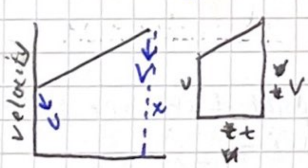
② $v = u + at$

③ $s = ut + \frac{1}{2}at^2$

④ $s = \frac{1}{2}(u+v)t$

- Equation Proofs

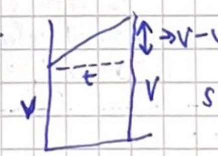
4. $s = \frac{1}{2}(u+v)t$



Area of trapezium $= \frac{h}{2}(a+b)$

$s = \frac{t}{2}(u+v)$

2.



so $a = \frac{v-u}{t}$

proving $v = u + at$

3. $s = ut + \frac{1}{2}at^2$

we know $s = \frac{1}{2}(u+v)t$ so plug this ② into 3.

$s = \frac{1}{2}(u + u + at)t$

$s = \frac{1}{2}(2ut + at^2)$

$s = ut + \frac{1}{2}at^2$

1. $v^2 = u^2 + 2as$

- we know $v = u + at$
- Rearrange $s = \frac{1}{2}(u+v)t$

$t = \frac{2s}{u+v}$

plug

$v = u + a\left(\frac{2s}{u+v}\right)$

$v^2 = u^2 + 2as$

Proving the Quadratic Formula

EX1:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $ax^2 + bx + c = 0$ $\div a$

$x^2 + \frac{bx}{a} + \frac{c}{a} = 0$ find CSF

$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$ so:

$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$

$\quad = \frac{b^2}{4a^2} \quad \frac{-4ac}{4a^2}$

$\quad = \frac{b^2 - 4ac}{4a^2}$

$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$

$x = -\frac{b}{2a} + \frac{\pm\sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Quadratic Graph

Q - If we have a graph of $y = 2x^2 + 5x - 1$ work out what line needs to be drawn to solve:

(a) $2x^2 + 4x - 1 = 0$

Add $1x$ to get to

$y = x$

(b) $2x^2 + 4x - 5 = 0$

$y = x + 4$

Graph of a Circle

EX1: Find where the line $x - y = 1$ meets the circle with the equation $x^2 + y^2 = 13$

$x = 1 + y$

$(1+y)^2 + y^2 = 13$

$(1+y)(1+y) + y^2 = 13$

$1 + 2y + y^2 + y^2 = 13$

$2y^2 + 2y - 12 = 0$ $\div 2$

$y^2 + y - 6 = 0$

$(y+3)(y-2) = 0$

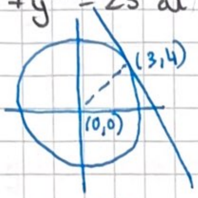
$y = -3$ and $y = 2$

$x = -2$ and 3

$(-2, -3)$ and $(3, 2)$

note: if line is tangent then only one answer.

EX2: Find an equation of the tangent to the circle $x^2 + y^2 = 25$ at the point $(3,4)$



• Since tangent is perpendicular to radius, just find gradient of tangent and make it perpendicular.

$m = \frac{\text{change } y}{\text{change } x} = \frac{4}{3}$

So $m_{perp} = -\frac{3}{4}$

NEW↓

$y - y_1 = m(x - x_1)$

$y - 4 = -\frac{3}{4}(x - 3)$ (can leave like this + get full marks!) Same as prev answer

OLD → $y = mx + c$

$y = -\frac{3}{4}x + c$

$4 = -\frac{3}{4} \times 3 + c$ $c = \frac{25}{4}$ so $y = -\frac{3}{4}x + \frac{25}{4}$ OR

READ BEHIND FIRST

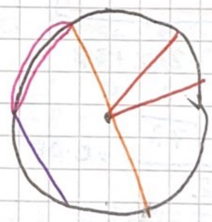
NEW FORMULA: Instead of $y = mx + c$ use: $\boxed{y - y_1 = m(x - x_1)}$

(x_1, y_1) is a point on the line

Area and Volume

Area of trapezium $= \frac{h}{2}(a+b)$

prism def → a 3D solid that has same cross-section all through it's length



Area of a circle $= \pi r^2$

Circumference → $\pi \times 2r / d$

Area of a sector $= x/360° \times \pi r^2$

Arc length $= x/360° \times 2\pi r$

volume of cylinder $= \pi \times r^2 \times h$

SA of cyllinder $= 2\pi r^2 + 2\pi r h$

volume of pyramid $= \frac{1}{3} \times$ base area $\times$ height

Volume of cone $= \frac{1}{3} \times (\pi \times r^2) \times$ height

curved SA of cone $= \pi \times r \times l$

Base SA of cone $= \pi \times r^2$

Vol of sphere $= \frac{4}{3} \times \pi \times r^3$

SA of sphere $= 4 \times \pi \times r^2$

BOUNDS

upper bound ⇒ $\frac{1}{2}$ a unit greater than

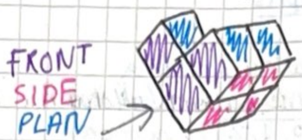
UB of fraction $= \frac{\text{UB of numerator}}{\text{LB of denominator}}$

lower bound ⇒ $\frac{1}{2}$ a unit less than

LB of a fraction $= \frac{\text{LB of } \cancel{\text{denominator}} \text{ numerator}}{\text{UB of denominator}}$

Elevations & Plans

tip! Begin by drawing front elevation



TRANSFORMATIONS, REFLEXIONS + ROTATIONS

transformation → moving one shape to a different position

reflexions → reflected on the other side of mirror line

rotations → same shape rotated around a point

• rotations

1. Direction of turn (clockwise, anticlockwise)
2. Angle of turn
3. Centre of rotation (where pencil pivots)

Put tracing paper out + copy shape, use pencil as a pivot and turn shape angle specified and draw shape

• Enlargement

1. type of transformation
2. scale factor
3. centre of enlargement

use 2 corners of shapes, where lines intersect = centre of Enlargement

when shape is enlarged by SF K, Area is enlarged K^2.

• Column vectors

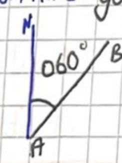
direction where point moves

Resultant vector - the vector that moves the OG shape to its final position after a no. of translations.

$\begin{pmatrix} +x \\ +y \end{pmatrix}$ → ↑ $\begin{pmatrix} -x \\ -y \end{pmatrix}$ ← ↓

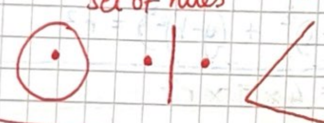
• Bearings

- measured CLOCKWISE given in 3 figures
- Northlines go up the page



Bisect → cut exactly in half

loci - set of points that obey a set of rules



Equations & Inequalities

• Finding roots of quadratic equations (harder)

a.) $x^2 - 2x$ $\quad x(x-2)$

$(x+0)(x-2)$

$x = 0$ or 2

b.) $4 - y^2$

$(y+2)(y-2)$

$y = 2$ or -2

c.) $x^2 - 16$

$(x-4)(x+4)$

$x = -4$ or 4

Ex: Find roots of $6x^2 - \cancel{23}x + \cancel{20}$ (5, −2)

a b c

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

roots of solve equation (2 solutions)

$$x = \frac{-5 + \sqrt{73}}{12} = 0.30$$

$$= \frac{-5 - \sqrt{73}}{12} = -1.13$$

• Completed Square form

$(x+a)^2 - b$

makes easier to solve quadratic especially for quadratics that don't factorise

Ex: Express $x^2 + 4x - 3$ in completed Square form where A and B are integers.

$x^2 + 4x - 3$

① $4 \div 2 = 2$ $\quad (x+2)^2$

② expand: $x^2 + 4x + 4$

③ get rid of extra: $(x+2)^2 - 4$

④ Remember −3!

$(x+2)^2 - 4 - 3 = (x+2)^2 - 7$

Ex2: Express $2x^2 + 4x + 1$ in completed square form

$2x^2 + 4x + 1$

① factorise

$2(x^2 + 2x) + 1$

② now complete square

$2 \div 2 = 1$

$2\left[(x+1)^2\right] + 1$

$(x+1)(x+1) = x^2 + 2x + 1$ so get rid of 1

$2\left[(x+1)^2 - 1\right] + 1$

③ get rid of [ ]

$2(x+1)^2 - 2 + 1 = 2(x+1)^2 - 1$

EX3: Solve the quadratic by completing the square. $x^2-4x-12=0$

$x^2-4x-12=0$

$-4\div 2=-2$

$(x-2)^2-4-12=0$

$\boxed{(x-2)(x-2) = x^2-4x+4}$

$(x-2)^2-16=0$

$(x-2)^2=16$

$x-2=4$ or $x-2=-4$

$\underline{x=6}$ $\underline{x=-2}$

Simultaneous equations

→ 2 Unknowns

→ 3 principles:

1. If we add 2 equations it still holds true
2. If we subtract 2 equations it still holds true
3. If we multiply the equation by any 'n' and add or subtract it still holds true

More on Completed Square form:

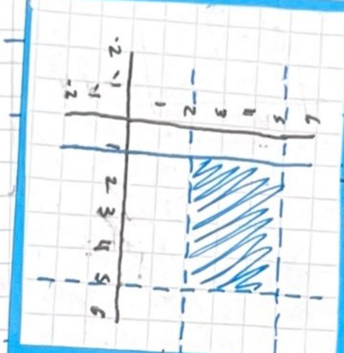
- $(4-x)^2=(x-4)^2$ (same!)
- In General for completed □ form: $y=(x-a)^2+b$

And since when $(x-a)^2=0$, $y=B$

we can deduce that the turning point is $(-a,b)$

Regions + Inequalities

Ex: Shade the region $1\leq x<5$ and $2<y<5$.



EX1: Find the maximum vertex of:

make it +

$y=6-2x-x^2$

$-x^2-2x+6$

$y=-1[x^2+2x-6]$

$y=-1[(x+1)^2-1-6]$

$y=-[(x+1)^2-7]$

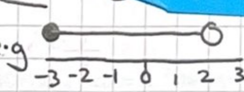
$y=7-(x+1)^2$

note: anything^2 is always +

turning point $(-1,7)$

turn me

e.g



$-3\leq n<2$

EX1: Solve $-7<2x+1<5$

$-7<2x+1$ $\quad$ $2x+1<5$

$-8<2x$ $\quad$ $2x<4$

$-4<x$ $\quad$ $x<2$

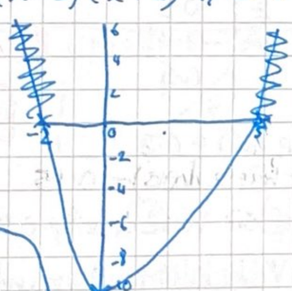
$=-4<x<2$

so...

$y=\pm -3-2\sqrt{2}$ or $-3+2\sqrt{2}$

EX2: $x^2-3x-10\geq 0$

$(x-5)(x+2)\geq 0$ so roots are 5 and -2



$y\geq 0$ so shade positive values

$y\leq -2$ or

$y\geq 5$

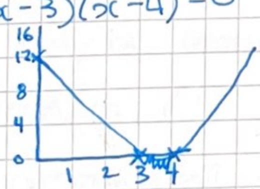
KEY RULE!

- if $x^2\leq 25$ then $= -5\leq x<5$
- if $x^2>36$ then $= x>6$ or -6

EX1: $x^2-7x+12\leq 0$ → y-intercept

#1 Factorise $(x-3)(x-4)=0$

#2 Sketch graph



#3 check if y

#4. $y\leq 0$ so shade negative region

#5. Write values

$3\leq x\leq 4$

Quadratics will never factorise

if $ax^2 + bx^2 + c = 0$

c = prime

Simultaneous equations

→ 2 Unknowns

→ 3 principles:

1. If we add 2 equations it still holds true
2. If we subtract 2 equations it still holds true
3. If we multiply the equation by any 'n' and add or subtract it still holds true

Alternative method for simultaneous equations

method 1 (+, −, × etc)

method 2 → Putting equation A into B

Ex: Find where lines $y = \frac{1}{x-2}$ and $xy + 2x = -7$ intersect.

Put A into B.

$x\left(\frac{1}{x-2}\right) + 2x = -7$

$\frac{x}{x-2} + 2x = -7 \qquad \times(x-2)$

$x + 2x(x-2) = -7(x-2)$

$x + 2x^2 - 4x = -7x + 14$

$2x^2 + 4x - 14 = 0 \qquad \div 2$

$x^2 + 2x - 7 = 0$

$(x+1)^2 - 1 - 7 = 0$

$(x+1)^2 = 8$

$x + 1 = \sqrt{8}$

$x = \pm\sqrt{8} - 1$ (2 answers)

so...

$y = -3 - 2\sqrt{2}$ or $-3 + 2\sqrt{2}$

= 14 × ...

$y = 14 \times 2$

= 70

= 28

= 42

$y = 42/9$

×6 +3

n×6

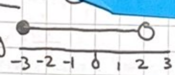
= −1

c + 3

Regions + Inequalities

Ex: shade the region $1 \leq x < 5$ and $2 < y < 5$.

e.g.



$-3 \leq n < 2$

EX1: Solve $-7 < 2x + 1 < 5$

$-7 < 2x + 1 \qquad 2x + 1 < 5$

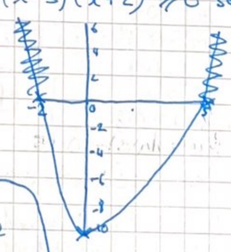
$-8 < 2x \qquad 2x < 4$

$-4 < x \qquad x < 2$

$= -4 < x < 2$

EX2: $x^2 - 3x - 10 \geq 0$

$(x-5)(x+2) \geq 0$ so roots are 5 and −2



$y \geq 0$ so shade positive values

$y \leq -2$ or

$y \geq 5$

Quadratic inequalities

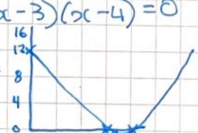
KEY RULE!

- if $x^2 \leq 25$ then $= -5 \leq x \leq 5$
- if $x^2 > 36$ then $= x > 6$ or -6

EX1: $x^2 - 7x + 12 \leq 0$ (+, ×, → y-intercept)

#1 Factorise $(x-3)(x-4) = 0$

#2 Sketch graph



#3 check if y value is less than/greater than 0 in given equation

#4. $y \leq 0$ so shade negative region

#5. Write values $3 \leq x \leq 4$

Quadratics will never factorise

if $ax^2 + bx^2 + c = 0$

c = prime

Simultaneous equations

→ 2 Unknowns

→ 3 principles:

① If we add 2 equations it sti

② If we subtract 2 equations it

③ If we multiply the equation and add or subtract it the

a) $4x - 3y = 7$
$+\ 2x - 3y = 2$

$6x = 9$

$x = 1.5$

$6 - 3y = 7$

$-3y = 1$

$3y = 1$

$y = 1/3$

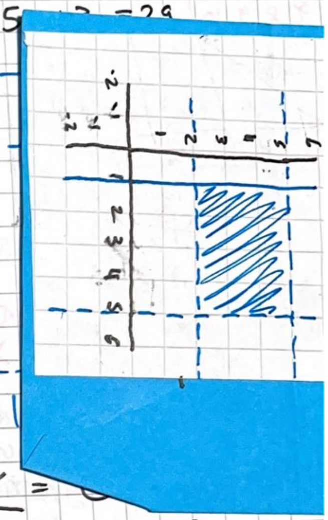
b) $2x + 3y = 14 \times 5$
$5x + 3y = 14 \times 2$

$10x + 15y = 70$
$-\ 10x + 6y = 28$

$9y = 42$

$y = 42/9$

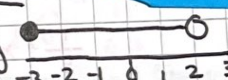
c) 5 ... = 29



o

< = o

e.g



$-3 \leq n < 2$

EX: Using simultaneous equations, find the equation of the line passing through $(6,-3)$ and $(-2,5)$

$y = mx + c$

$-3 = m6 + c$

$5 = m-2 + c \quad \times 3$

$-3 = m6 + c$

$+\ 15 = m-6 + 3c$

$12 = 4c$

$c = 3$

$-3 = m \times 6 + 3$

$-6 = m \times 6$

$m = -1$

hence,

$y = -x + 3$

EX1: Solve $-7 < 2x + 1$ ...

$-7 < 2x + 1$ $2x$ ...

$-8 < 2x$

$-4 < x$

$= -4 < x <$ ...

o Quadratic ~~equations~~ inequalities

KEY RULE!

• if $x^2 \leq 25$ then $= -5 \leq x \leq 5$

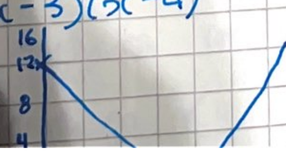
• if $x^2 > 36$ then $= x > 6$ or -6

EX1: $x^2 - 7x + 12 \leq 0$

$+ \quad \times$ → y-intercept

#1 Factorise $(x-3)(x-4) = 0$

#2 Sketch graph



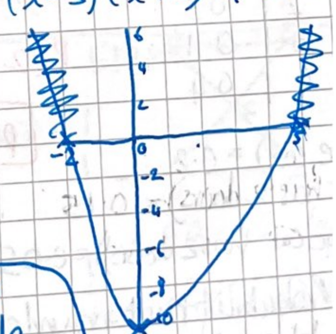
#4. $y \leq 0$ so shade negative region

#5. Write values

$3 \leq x \leq 4$

EX2: $x^2 - 3x - 10 \geq 0$

$(x-5)(x+2) \geq 0$ so



PROBALITY

- when m ways of doing x and n ways of doing y the total number of ways to do x and y is $m \times n$ (explained more at point 3)

- Sample space diagram Ex1. 2 fair 5 sided ~~dice~~ spinners are spun and the results are added together
(a) draw a sample space diagram to show possible outcomes

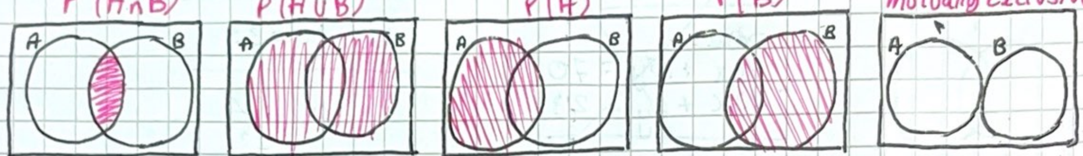
0	1	2	3	4	5
1	2	3	4	5	6
2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10

(b.) Find $P(6) = \frac{5}{25} = \frac{1}{5}$

(c) Find $P(\text{Prime}) = \frac{11}{25}$

$P(A/B)$ = Probability of A given B has occured

- P(A∩B) P(A∪B) P(A) P(B) mutually exclusive



AND OR

AND - independent

Provided 2 events are independent (one event occuring doesn't affect the likelihood of another event occuring) we $\times$ the probability of A and B occuring.

A∩D

Ex:2 $P(\text{Sunny tomorrow}) = \frac{1}{5}$ $P(\text{win lottery card}) = \frac{1}{20}$

so $P(\text{sunny tom} \cap \text{win lottery})$

$\frac{1}{5} \times \frac{1}{20} = \frac{1}{100}$

OR - Mutually exclusive

2 events are mutually exclusive when 1 outcome of 1 excludes the possibility of the outcome of the other (cannot have same outcomes @ same time)

EX3: P(raining) excludes P(Not raining) at same time.

EX4:

Points	Probability
1	0.2
2	0.3
3	0.1
4	0.4

This table shows 4 different outcomes on a 4 spinner.
Find $P(2 \text{ Points} \cup 3 \text{ Points})$

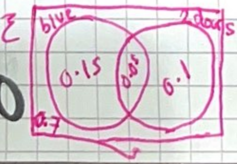
$P(2 \cup 3) = 0.3 + 0.1 = 0.4$

$\boxed{P(A) + P(B) - P(A \cap B) = P(A \cup B)}$

- EX5: $P(\text{blue car}) = 0.2$
$P(\text{car has 2 doors}) = 0.15$
$P(\text{blue car has 2 doors}) = 0.05$

Find the Probability that a randomly chosen car is blue or has 2 doors.

! Can't Assume it's independent or mutually exclusive



$P(A) + P(B) - P(A \cap B) = P(A \cup B)$

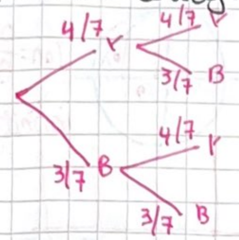
$0.15 + 0.1 - 0.05 = P(A \cup B)$

$P(A \cup B) = 0.2$

- Tree diagrams WITH replacement

EX6:
A bag contains 4 yellow counters and 3 blue counters. A counter is removed, it's colour noted, then replaced. A second counter is then removed and it's colour noted.

(i) Draw a tree diagram



(ii) Find the P(removing 2 counters of same colour)

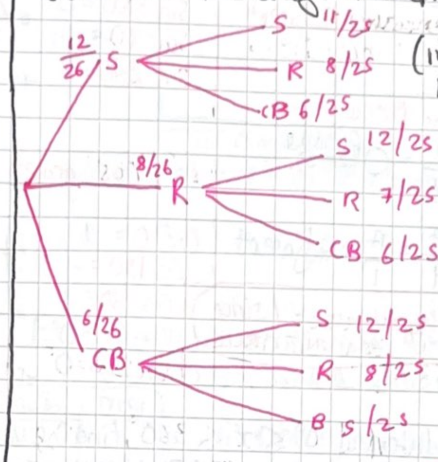
$\frac{4}{7} \times \frac{4}{7} = \frac{16}{49}$

$\frac{3}{7} \times \frac{3}{7} = \frac{9}{49}$

$\frac{16}{49} + \frac{9}{49} = \frac{25}{49}$

- Tree diagrams WITHOUT replacement

EX7:
There are: 12 Sherbets 8 Refreshers 6 Choc button in a bag.
2 sweets are selected at random
(i) Draw a tree diagram to represent this



(ii) Find the P(1 eat 2 sweets that are same)

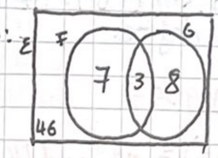
$\left(\frac{12}{26} \times \frac{11}{25}\right) + \left(\frac{8}{26} \times \frac{7}{25}\right) + \left(\frac{6}{26} \times \frac{5}{25}\right) = \frac{109}{325}$

- Conditional Probability - When you are given Xtra info which "shrinks" the sample space.

$$\boxed{P(A \mid B) = \frac{P(A \cap B)}{P(B)} \qquad P(B \mid A) = \frac{P(A \cap B)}{P(A)}}$$

AKA when events are dependent on one another.

EX8:



venn represents students who A level choices.

Find the Probability that a randomly selected student...

(i) Studies German

$P(G) = \frac{11}{64}$

(ii) Studies German and French

$P(F \cap G) = \frac{3}{64}$

(iii) Find the Prob that they study French given that they studied German.

$P(F) = 10/64$

$P(F \mid G) = 3/64$

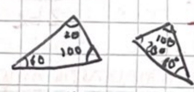
(iv) Find the Prob that they study German given that they don't study French.

$P(G \mid F') = \frac{8}{64}$

CONGRUENCY

↳ identicle in both SHAPE and SIZE

1 A A A ~~SVSSS~~



→ similarity proof not always congruent

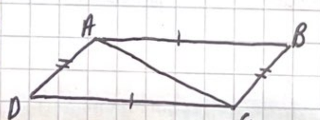
2 S A S (Angle must be between 2 sides no ASS!)

3 A S A (Side doesn't have to be between angles)

4 R H S

5 S S S

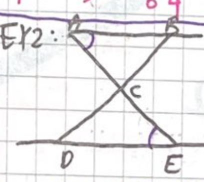
EX1: ABCD is a parallelogram. prove That △ABC is congruent to △ADC.



- length AD = length CB (opposites in a parallelogram are =) (S)
- length AB = length CD (" " " ") (S)
- length AC is common to both (S)

Hence congruent by SSS

EX2:



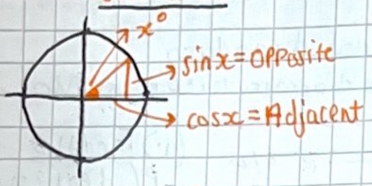
Given that AB = DE prove that △ABC is congruent to △CDE.

- AB = DE (given) (S)
- $C\hat{A}B = C\hat{E}D$ (alternate angles) (A_1)
- $A\hat{B}C = C\hat{D}E$ (" ") (A_2)

Hence congruent by ASA.

TRIGONOMETRY #2

• **Unit Circle**



Because...

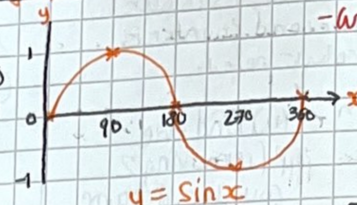
$\sin x = \frac{O}{H} = O$ Opposite

$\cos x = \frac{A}{H} = \frac{A}{1} = $ Adjacent

note ↑ → hypotenuse = 1 since it's a UNIT CIRCLE

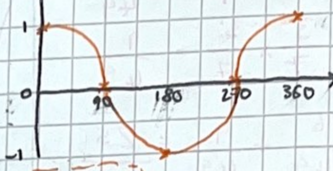
• **Sine graph**

Sin 0 = 0
Sin 180 = 0
Sin 360 = 0
Sin 90 = 1
Sin 270 = −1



• **Cos graph**

Cos 0 = 1
Cos 180 = −1
Cos 360 = 1
Cos 90 = 0
Cos 270 = 0

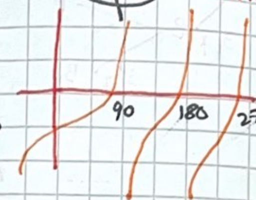


Tangent function

– when we get 90° the value of x, we say that @ 90° we have an asymptote.

$Tan = \frac{O}{A}$

infinity



EX1: In the interval $0° \leq x \leq 360°$, find the values of x for which $\sin x = 0.9397$. Give your answer to the nearest degree.

$\sin(x) = 0.9397$

$\sin^{-1}(0.9397) = 70°$ and $110°$

• **Area of a triangle with sine**

$\frac{1}{2} \times ab \times \sin \times C°$

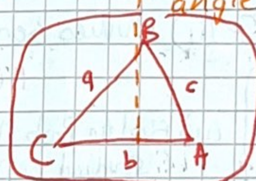
ab ↑ any 2 sides; C° → angle between

sine rule

• We use the sine rule if there are 2 opposite pairs

(looking for Angle) $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

(looking for a side) $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$



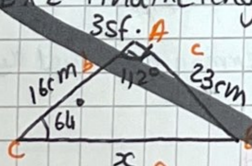
Cosine rule

• We use the cosine rule when we have 2 sides and angle in between (included angle)

(lengths) $a^2 = b^2 + c^2 - 2bc\cos A$

(Angle) $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

EX2: Find the length of side x. Give ans to 3sf.



~~$x^2 = 16^2 + 23^2$~~

$\frac{x}{\sin(112)} = \frac{23}{\sin(64)}$

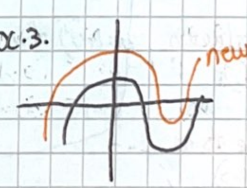
$\frac{x}{\sin(112)} = 25.58...$

$x = 23.6$ cm

LOOK @ Pg 22

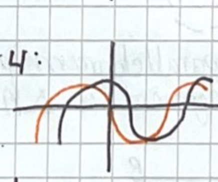
Transforming Trigonometric graphs

EX3:



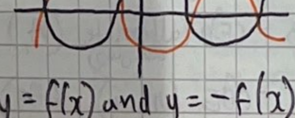
$y = f(x)$ and $y = f(x) + a$ is a translation by the vector: $\begin{bmatrix} 0 \\ a \end{bmatrix}$ ← y axis, x axis

EX4:



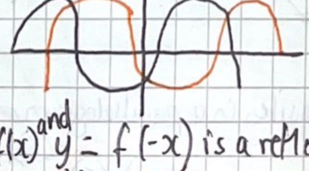
$y = f(x)$ and $y = f(x + a)$ is a translation by vector: $\begin{bmatrix} a \\ 0 \end{bmatrix}$

EX5:



$y = f(x)$ and $y = -f(x)$ is a reflection in x axis.

EX6:



$y = f(x)$ and $y = f(-x)$ is a reflection in y-axis.

MORE ON SINE-COSINE RULE PG22

Population + Stratified Sampling

POPULATION – A set of items you are interested in

CENSUS – A survey on the whole population

Ex1: I catch 60 fish and tag them. Next day I catch 100 fish, 20 are tagged. Estimate how many fish are in the lake

To estimate the size of population, N of species;
- capture + mark a size N
- Recapture another sample size M

$\frac{n}{N} = \frac{m}{M}$ so, $N = \frac{n \times M}{m}$

Ex1 answer

$\frac{60}{N} = \frac{20}{100}$ so $N = \frac{60 \times 100}{20}$

$= 300$ fish

• St

• To
A

(eg IQR)

Compare → Compare ① and ②

2 Key Ideas ↓

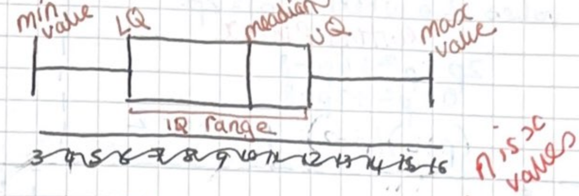
① IQR measures spread of middle 50% of data

② meadian and IQR are NOT affected by extreme values or outliers

Cumulative frequency + Box plots

→ What does a cumulative frequency table show

how many data values are less than or equal to the UB of each data set. (on y-axis) create running total.



n is no values

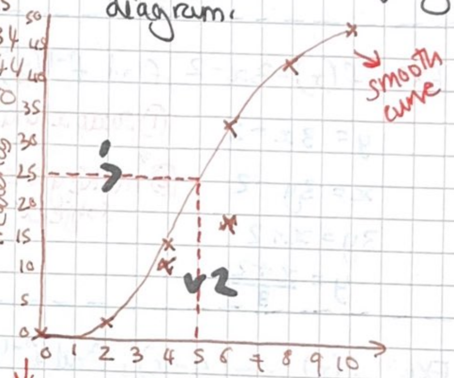
$LQ = \frac{n+1}{4}$ $UQ = \frac{3(n+1)}{4}$

IQ range = UQ − LQ

EX1: This table gives the times taken by 50 students to solve a Maths puzzle.

(minutes)	Freq	
≤ 2	3	3
≤ 4	12	15
≤ 6	19	34
≤ 8	10	44
≤ 10	6	50

(i) Draw a cumulative frequency diagram.



Plot (0,0)

Plot each frequency at UB of each data set.

(ii) Use graph to find the meadian

draw line from half way of cumulative Freq

= 25

meadian = 5

(iii) Estimate the range of time

10 − 0 = 10

(iv) IQR = 8.25 − 2.75 = 5.5

UQ = 3(10+1) ÷ 4 = 8.25

LQ = 10+1 ÷ 4 = 2.75

EX2:

weight	freq	CF
0 < W ≤ 20	10	10
20 < W ≤ 30	15	25
30 < W ≤ 50	18	43
50 < W ≤ 60	13	56
60 < W ≤ 75	15	71
75 < W ≤ 100	10	81

Calculate an estimate for the IQR of weights of oranges.

Meadian = 40.5

Similar to Estimating Mean of histogram

LQ = m ÷ 2 = 40.5 ÷ 2 = 20.25

UQ = LQ + m = 40.5 + 20.25 = 60.75

LQ

1. 20.25 − 10 = 10.25

2. $\frac{10.25}{15} \times 10 + 20$ = 26.8

UQ

1. 60.75 − 56 = 4.75

2. $\frac{4.75}{15} \times 15 + 60$ = 65.5

① LQ/UQ from prev group

② $\frac{\text{Ans}}{\text{freq of group}} \times$ Class Width $\times$ weight up to that

IQ Range = 65.5 − 26.8 = 38.7

Quadratic graphs

• lowest / highest point of parabola = turning point
• To find turning point coordinates, write in completed Square form

$y = a(x+b)^2 + c$

POP ... ed in

CEN ... tion

Ex1: ... ay I ... te

What assumptions have you made?
1. population hasn't changed
2. marked fish are evenly distributed in the lake
3. tags have not come off
4. P(catching fish) is the same

• Stratified Sampling

$$\frac{\text{sample size}}{\text{Population size}} \times \text{stratum size (group)}$$

Cumulative frequency + Box plots

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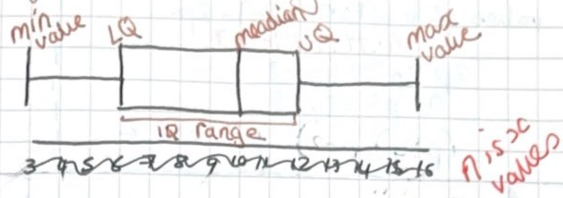
how many data values are less than or equal to the UB of each data set. (on y-axis) create running total.



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$8 < t \leq 10$	6	50

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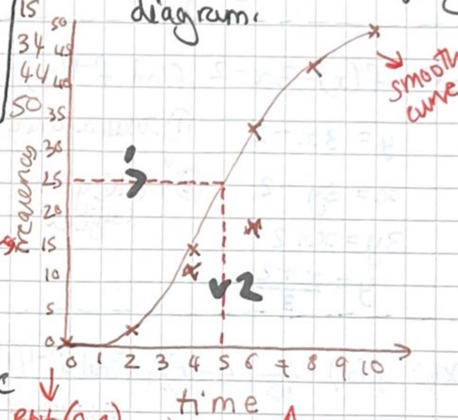

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draw line from halfway of cumulative Freq
= 25
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UQ = 3(10+1) ÷ 4 = 8.25
LQ = 10+1 ÷ 4 = 2.75

• TO DESCRIBE + COMPARE A POPULATION:

Describe → ① measure of average (eg meadian)
② measure of spread (eg IQR)

Compare → Compare ① and ②

2 Key Ideas

① IQR measures spread of middle 50% of data

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EX2:

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LQ
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UQ
1. 60.75 − 56 = 4.75
2. $\frac{4.75}{15} \times 15 + 60$ = 65.5

① LQ/UQ from prev group
② $\frac{\text{Ans}}{\text{freq of group}} \times \text{Class Width} \times \text{weight up to that}$

IQ Range = 65.5 − 26.8 = 38.7

Quadratic graphs

• lowest / highest point of parabola = turning point
• To find turning point coordinates, write in completed square form
$y = a(x+b)^2 + c$

Functions & iteration

- $f(x) = 2x^2$ same as $y = 2x^2$
  → means to get from x to y

- EX1: If $f(x) = x^2 + 4x - 8$, find the values of p when $f(p) = 2p$.

  Putting P into x

  $2p = p^2 + 4p - 8$
  $0 = p^2 + 2p - 8$
  $(p+4)(p-2)$ $p = -4$ or 2

EX2: Find the function f so that $f(0) = 3$, $f(1) = 8$ and $f(2) = 13$.

$f(0) = 3 \rightarrow 0 \, 6$
$f(1) = 8$) +5
$f(2) = 13$) +5

$f(x) = 5x + 3$

EX3: $f(x) = 3x - 2$, find $f^{-1}(x)$

$y = 3x - 2$ ① swap x and y
$x = 3y - 2$ ② make y the subject
$3y = x + 2$
$y = \frac{x+2}{3}$

EX4: $f(x) = \frac{3}{10}(5 - x)$, find $f^{-1}(6)$

① when given a number just write it out

$6 = \frac{3}{10}(5 - x)$
$20 = 5 - x$
$x = -15$

Ex 5: Functions are defined as $f(x) = \frac{x}{7} + 3$ and $g(x) = \frac{x}{3} + 7$

Prove that $f^{-1}(x) - g^{-1}(x) = 4x$

$f^{-1}(x) = \frac{y}{7} + 3$ | $g^{-1}(x) = \frac{y}{3} + 7$
$x - 3 = y \div 7$ | $y = 3x - 21$
$7x - 21 = y$

→ always remember brackets

$(7x - 21) - (3x - 21) = 4x$

$7x - 21 - 3x + 21 = 4x$
$4x = 4x$
proven ✓

14

x^2 superscript x_2 subscript

- Composite functions

EX 6: $f(x) = \frac{1}{x}$ $g(x) = 2x + 1$

$fg(x)$

Read from right to left.
Put 6 into f

$fg(x) = \frac{1}{2x+1}$

- Trial + Improvement

(guessing)

EG7: Find the solution to $x^3 + x = 7$ to 2dp using trial + improvement.

guess x	answer $x^3 + x = 7$	comments
1	2	too small
2	10	to big
1.5	4.875	too small
1.7	6.613	too small
1.75	7.109	Just too big
1.74	7.008	very close
1.73	6.907717	too small

next step vital to getting full marks. test middle

1.735 is just small so round to 2dp $= \underline{1.74}$

→ depends on decimal points

Interval Bisection Method

Solve $x^2 + 4x - 325 = 0$ to 1dp.

Let $f(x) = x^2 + 4x - 325$
$f(20) = 20^2 + 80 - 325 = 155$ [too big]
$f(15) = 15^2 + 80 - 325 = -400$ [too small]
so $15 < x < 20$
$f(17) = 32$ $f(16) = -5$
so $16 < x < 17$
$f(16.5) = 13.25$
so $16 < x < 16.5$
continue to until it becom same dp.

Subscript EX Question

EX6: If $x_{n+1} = 3x_n + 1$ and $x_1 = 2$. Find x_2.

$x_2 = 3 \times x_1 + 1$
$x_2 = 6 + 1 = 7$

Find x_3

$x_3 = 3 \times x_2 + 1$
$x_3 = 21 + 1 = 22$

Iteration to solve Quadratics

Ex: $y = x^2 + x - 5$ to 5dp

1/2 ① Rearrange to make x the subject

$0 = x^2 + x - 5$ $x^2 = x - 5$

$x_1 = \sqrt{5 - x_0}$

② Find pattern

$x_{n+1} = \sqrt{5 - x_n}$

③ Keep changing + pressing = key until value repeats

EX: Apply $x_{n+1} = \sqrt{325 - 4x_n}$ with $x_1 = 13$ to find a root of $x^2 + 4x - 325 = 0$ correct to 2dp.

$x_{n+1} = \sqrt{325 - (4 \times 13)}$

$x_{n+1} = \sqrt{325 - 52} = \sqrt{273}$

$x_{n+1} = 16.52...$

$x_2 = 16.52...$

$x_3 = 16.09...$

$x_4 = 16.14...$

$x_5 = 16.13...$

eg here its 13

initial value

CALC STEPS
1 Press in number
2. Press =
3. use $\boxed{\text{ANS}}$ key

$= \underline{16.14}$

$f(x) = x^2 + 2x + 1$

sw that $f(x+2) - f(x) = 4x + 8$

$(x+2) = (x+2)^2 + 2(x+2) + 1$

$= x^2 + 6x + 9$

$(x^2 + 6x + 9) - (x^2 + 2x + 1)$

$= 4x + 8$

prooven ✓

vectors and geometric proof

magnitude - length (e.g length, Area, volume, pressure, time) $\sqrt{x^2+y^2}$

displacement - change in position (e.g displacement, velocity, acceleration)

- The displacement vector from A to B is written $\overrightarrow{AB}$ or $\underset{\rightarrow}{a}$ $\underset{\rightarrow}{b}$ $\underset{\sim}{c}$

$|a|$ means magnitude of vector $\underset{\rightarrow}{a}$

$\begin{pmatrix} + - x \\ + - y \end{pmatrix}$

EX1: A is (3,4) and B is (-3,0)

(i) Write $\overrightarrow{AB}$ as a column vector

(A − B) $\begin{pmatrix} -6 \\ -4 \end{pmatrix}$

(ii) Find the length of vector $\overrightarrow{AB}$

$\sqrt{x^2+y^2}$ $\sqrt{-6^2-4^2}$

$= 2\sqrt{13}$

~~OOPS WROTE TWICE~~

EX2: Add vectors a and b to get resultant c.

$a = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ $b = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ $\begin{pmatrix} -2 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$

CAN'T MULTIPLY VECTORS!

b.) draw a nose to tail diagram to show that the answer makes diagramatic sense.

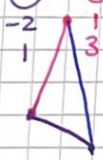
(1) choose point for a

(2) draw b from end point of a

(3) finally draw c, which should tie lines

a b c

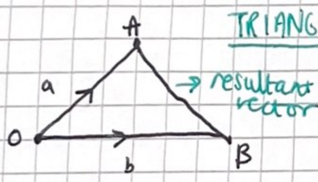
$\begin{matrix} -2 \\ 1 \end{matrix}$ $\begin{matrix} 1 \\ 3 \end{matrix}$ $\begin{matrix} -1 \\ 4 \end{matrix}$



- $\overrightarrow{AB} = 8a + 20b$
  $\overrightarrow{CD} = 2a + 5b$

position vectors

TRIANGLE LAW FOR VECTOR ADDITION



~~If you X 1 vector by another vector~~

- If 1 vector is the multiple of another, they are parallel

In other words:

2 vectors are parallel when one is a scalar multiple of another.

e.g $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ $\begin{pmatrix} -4 \\ 6 \end{pmatrix}$
$\quad\underset{\sim}{a}\quad\quad\underset{\sim}{b}$

$-2\underset{\sim}{a} = \underset{\sim}{b}$

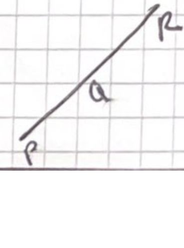
steps taken to prove 3 points lie on the same line → called co-linear

1. Make 2 vectors out of them
2. State they share a common point

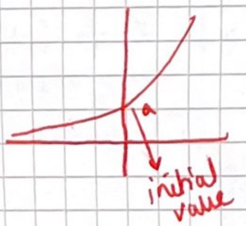
- If 2 vectors are parallel and share the same point, they are co-linear

e.g ~~PQR~~ ~~PR~~ ~~QR~~

$\overrightarrow{PQ}$ $\overrightarrow{QR}$



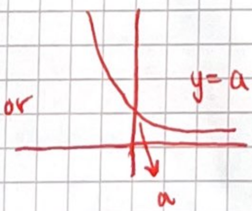
Exponential Problems



example

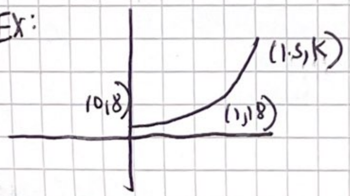
$y = a \times b^x$

or



$y = a \times b^{-x}$

Ex:



$y = pq^x$ (0, 8)

$8 = p \times q$

$\underline{p = 8}$

$y = 8 \times q^x$

$\underline{q = 9 \div 4}$

$K = \left(\frac{9}{4}\right)^{3/2} \times 8$

$K = 27$

→ vectors continued

To find the length of a vector: → AKA $|AB|$

• $A = (2, -3)$ $B = (6, 4)$ so $\vec{AB} = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$

$\begin{pmatrix} 4 \\ 7 \end{pmatrix}$ $|AB| = \sqrt{4^2 + 7^2} = \sqrt{65}$

Multiplication by a scalar

• Let $\underset{\sim}{a} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$

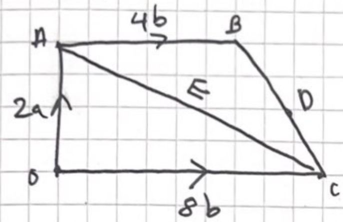
what is $7a + 2b$?

$7\begin{pmatrix} 3 \\ 2 \end{pmatrix} + 2\begin{pmatrix} 5 \\ -3 \end{pmatrix}$

$\begin{pmatrix} 21 \\ 14 \end{pmatrix} + \begin{pmatrix} 10 \\ -6 \end{pmatrix} = \begin{pmatrix} 31 \\ 8 \end{pmatrix}$

EG 1: OABC is a trapezium.
Point D is the midpoint of BC.
Point E is the midpoint of AC.

$\vec{OA} = 2a$, $\vec{AB} = 4b$ and $\vec{OC} = 8b$



a.) write out these vectors in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$

(i) $\vec{OB} = 2a + 4b$

(ii) $\vec{AC} = -2a + 8b$

(iii) $\vec{AE} = \frac{1}{2}\vec{AC}$

$= \frac{1}{2}(-2\underset{\sim}{a} + 8\underset{\sim}{b})$

$= 4\underset{\sim}{b} - \underset{\sim}{a}$

b.) Show $\vec{ED}$ and $\vec{OC}$ are parallel.

$\vec{OC} = 8\underset{\sim}{b}$

$\vec{ED} = \vec{EC} + \vec{CD}$

we know $\vec{EC} = \vec{AE}$ (midpoint)

$= (4\underset{\sim}{b} - \underset{\sim}{a}) + \vec{CD}$

$(4\underset{\sim}{b} - \underset{\sim}{a}) + \frac{1}{2}(\vec{CB})$

$(4b - a) + \frac{1}{2}(-8b + 2a + 4b)$

$\frac{1}{2}(-4b + 2a)$

$(4b - a) + -2b + a$

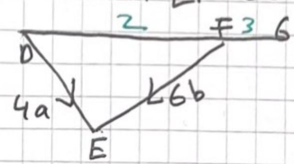
$= \underline{\underline{2b}}$

As $4\vec{ED} = \vec{OC}$ (they are scalar multiples of one another) they are parallel.

EG 2.

DFG is a straight line

$\vec{DE} = 4a$ and $\vec{EF} = 6b$.



DF : FG = 2 : 3

Work out the vector $\vec{EG}$ giving your answer in its simplest form.

$\vec{EG} = \vec{EF} + \vec{FG}$

use ratio

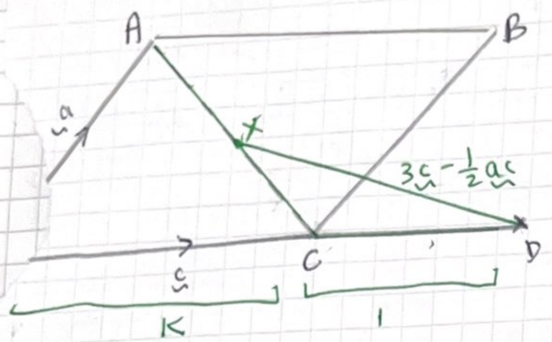
$= 6b + 3\left(\frac{DF}{2}\right)$

$\vec{EG} = 6b + 3\left(\frac{4a + 6b}{2}\right)$

$\vec{EG} = 6b + 6a + 9b$

$= 15b + 6a$

HARDEST EVER VECTOR QUESTIONS ON REVERSE SIDE



OABC is a parallelogram

$\vec{OA} = a$ $\vec{OC} = c$

X is the midpoint of line AC.

OCD is a straight line so that OC : CD = K : 1

Given that $\vec{XD} = 3c - \frac{1}{2}a$

Find the vale of k.

$\vec{OD} = \frac{K+1}{k} c$

need to find another route for $\vec{OD}$

$\vec{OD} = \vec{OA} + \vec{AX} + \vec{XD}$

$\vec{AC} = -a + c$

$\vec{AX} = \frac{1}{2}(-a + c)$

$\vec{OD} = a + \frac{1}{2}(-a+c) + 3c - \frac{1}{2}ac$

$= a + -\frac{a}{2} + \frac{c}{2} + 3c - \frac{1}{2}ac$

$0.5a + 3.5c - 0.5a - 0.5c$

$= 3.5c$

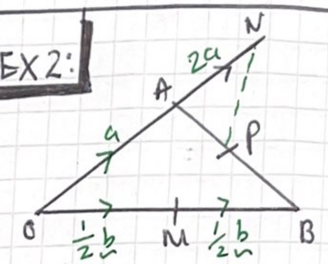
so $\frac{K+1}{K} = 3.5c$

$K + 1 = 3.5K$

$1 = 2.5K$

$k = \frac{1}{2.5} = \frac{2}{5}$

EX2:



OAN, OMB and APB are straight lines

AN = 2OA

M is the midpoint of OB

$\vec{OA} = a$ $\vec{OB} = b$

$\vec{AP} = K\vec{AB}$ where K is a scalar quantity.

Given that MPN is a straight line, find the vale of K.

$\vec{AB} = -a + b$

$\vec{AP} = K(-a + b)$

other way to get ~~AB~~ $\vec{AP}$

$\vec{AP} = \vec{AN} + \vec{NP}$

EQUATING COEFFICIENTS

$\vec{AP} = 2a + \boxed{}$

$\vec{NM} = -3a + \frac{1}{2}b$

let $\vec{NP} = d\vec{NM}$ [where d is scalar] (new variable)

$\vec{NP} = d \times (-3a + \frac{1}{2}b)$

so $\vec{AP} = 2a + d(-3a + \frac{1}{2}b)$

$\vec{AP} = a(2-3d) + b\left(\frac{d}{2}\right)$

$\vec{AP} = a(-K) + b(K)$

$\vec{AP} = a(2-3d) + b\left(\frac{d}{2}\right)$

$-K = 2 - 3d$

~~K = 2~~

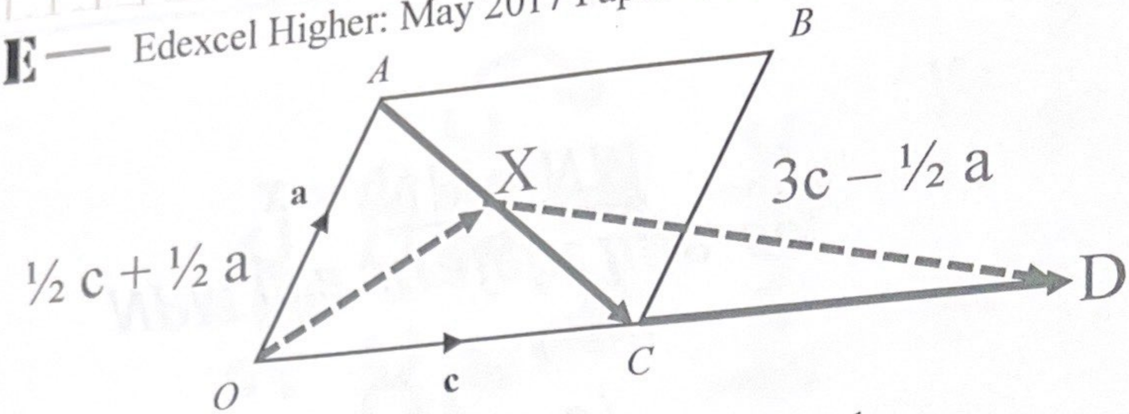
$2K = d$

(substitute) get rid of d $-K = 2 - 3(2K)$

$K = 2/5$

GCSE — Edexcel Higher: May 2017 Paper 1, Q19

1



1

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K

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X is the midpoint of the line AC.
OCD is a straight line so that $OC : CD = k : 1$

Given that $\overrightarrow{XD} = 3\mathbf{c} - \frac{1}{2}\mathbf{a}$ find the value of k.

$\overrightarrow{AC} = c - a$ $\quad \overrightarrow{AX} = \frac{1}{2}(c - a) = \frac{1}{2}c - \frac{1}{2}a$

$\overrightarrow{OX} = a + (\frac{1}{2}c - \frac{1}{2}a) = \frac{1}{2}c + \frac{1}{2}a$

$\overrightarrow{OD} = \overrightarrow{OX} + \overrightarrow{XD} = \frac{1}{2}c + \frac{1}{2}a + 3c - \frac{1}{2}a$

$= 3\frac{1}{2}c$

$\overrightarrow{CD} = 2\frac{1}{2}c$

$1 : 2.5$

$\frac{1}{2.5} : 1 \quad (\div 2.5)$

$\frac{10}{25} : 1$

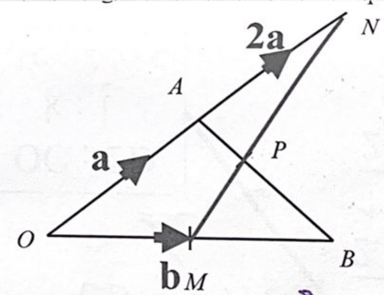
$\frac{2}{5} : 1$

$$\boxed{\begin{array}{c} OC : CD \\ k : 1 \\ 1c : 2\frac{1}{2}c \end{array}}$$

$k = \frac{2}{5}$

(Total for Question 1 is 4 marks)

1



Alternate workings

OAN, *OMB* and *APB* are straight lines.
$AN = 2OA$.
M is the midpoint of *OB*.

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

$\overrightarrow{AP} = k\overrightarrow{AB}$ where k is a scalar quantity.

Given that *MPN* is a straight line, find the value of k.

$\overrightarrow{AB} = -a + b$

$\overrightarrow{MN} = 3a - 0.5b$

$\overrightarrow{MP} = -0.5b + a - ka + kb$

$\overrightarrow{NM} = -3a + 0.5b$

$\overrightarrow{NP} = -2a + k(-a+b)$

NPM is a straight line so...

$x \times NP = NM$

$k = 2/5$

$$\overrightarrow{AB} = -a + b$$

$$\overrightarrow{AP} = k(-a + b)$$

$$\overrightarrow{NM} = -3a + \tfrac{1}{2}b$$

$$\overrightarrow{NP} = -2a + k(-a + b)$$

NPM is a straight line, so,

$$x \times \overrightarrow{NP} = \overrightarrow{NM}$$

$k = \frac{2}{5}$ ....

(Total for Question 1 is 5 marks)

$$x(-2a + k(-a + b)) = -3a + \tfrac{1}{2}b$$

$$x(-2a - ka + kb) = -3a + \tfrac{1}{2}b$$

$$-2xa - kxa + kxb = -3a + \tfrac{1}{2}b$$

Split coefficients

a	b
$-2x - kx = -3$	
$2x + kx = 3$	$kx = \frac{1}{2}$

Substitute to eliminate x

$x = \frac{1}{2k}$

$$2\left(\frac{1}{2k}\right) + k\left(\frac{1}{2k}\right) = 3$$

$$\frac{2}{2k} + \frac{k}{2k} = 3$$

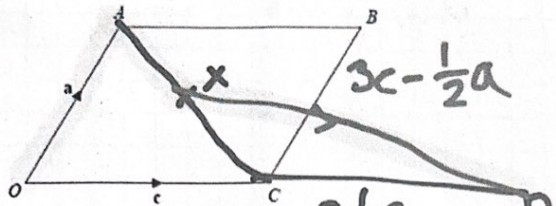
$$\frac{1}{k} + \frac{1}{2} = 3$$

$$\frac{1}{k} = 2.5 = \frac{5}{2}$$

$$k = \frac{2}{5}$$

5 HARDEST VECTOR QUESTIONS

19



$\vec{CD} = 2\frac{1}{2}c$

OC : CD

$c : 2\frac{1}{2}c$

$\div 2\frac{1}{2}$ $\frac{2}{5} : 1$ $\div 2$

OABC is a parallelogram.

$\vec{OA} = \mathbf{a}$ and $\vec{OC} = \mathbf{c}$

X is the midpoint of the line AC.

OCD is a straight line so that OC : CD = k : 1

Given that $\vec{XD} = 3\mathbf{c} - \frac{1}{2}\mathbf{a}$

find the value of k.

$c \quad : \quad 2\frac{1}{2}c$

$\vec{AC} = -a + c$

$\vec{Ax} = \frac{1}{2}(-a+c) = -\frac{1}{2}a + \frac{1}{2}c$

$\vec{Ox} = a - \frac{1}{2}a + \frac{1}{2}c$

$= \frac{1}{2}a + \frac{1}{2}c$

$K = 2/5$

$1 \div 2\frac{1}{2}$

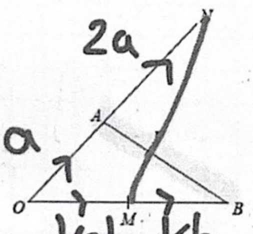
$1 \div \frac{5}{2}$

$1 \times \frac{2}{5} = \frac{2}{5}$

$\vec{OD} = \frac{1}{2}a + \frac{1}{2}c + 3c - \frac{1}{2}a = 3\frac{1}{2}c$

10:17 / 42:37

21



OAN, OMB and APB are straight lines.

AN = 2OA.

M is the midpoint of OB.

$\vec{OA} = \mathbf{a}$ $\quad \vec{OB} = \mathbf{b}$

$\vec{AP} = k\vec{AB}$ where k is a scalar quantity.

Given that MPN is a straight line, find the value of k.

$\vec{AB} = b - a$

$\vec{MN} = 3a - \frac{1}{2}b$

$\vec{MP} = \vec{MO} + \vec{OA} + \vec{AP}$ $\quad = k\vec{AB}$

$= -\frac{1}{2}b + a + k(b-a)$

$= -\frac{1}{2}b + a + kb - ka$

$= a - ka - \frac{1}{2}b + kb$

$= a(1-k) - b(\frac{1}{2} - k)$

Coefficient of a must be 6 times greater than coefficient of B

$1 - k = 6(\frac{1}{2} - k)$

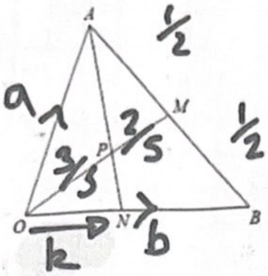
$1 - k = 3 - 6k$

$1 + 5k = 3$

$5k = 2$

$k = 2/5$

21



ON : NB

$\frac{3}{7} : \frac{4}{7}$

$3:4$

OAB is a triangle.
OPM and APN are straight lines.
M is the midpoint of AB.

$\vec{OA} = \mathbf{a} \quad \vec{OB} = \mathbf{b}$

$OP : PM = 3 : 2$

Work out the ratio $ON : NB$

$\vec{AB} = b - a$

$\vec{AM} = \frac{1}{2}b - \frac{1}{2}a$

$\vec{OM} = a + \frac{1}{2}b - \frac{1}{2}a$

$= \frac{1}{2}a + \frac{1}{2}b$

$\vec{AP} = -a + \frac{3}{5}\left(\frac{1}{2}a + \frac{1}{2}b\right)$

$= -a + \frac{3}{10}a + \frac{3}{10}b \qquad \vec{AN} = -a + k(b)$

$\vec{AP} = \frac{3}{10}b - \frac{7}{10}a \qquad x\vec{AP} = \vec{AN}$

$= \frac{1}{10}(3b - 7a) \qquad \frac{x}{10}(3b - 7a) = -a + k(b)$

$\frac{3x}{10}b - \frac{7x}{10}a = -a + k(b)$

$\frac{3x}{10} = k \qquad -\frac{7x}{10} = -1$

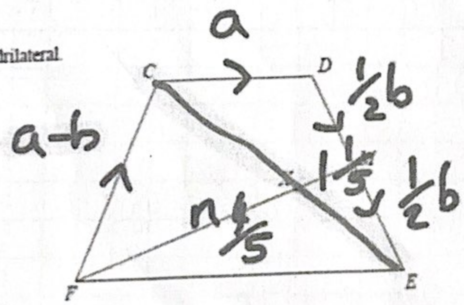
$3x = 10k \qquad 7x = 10$

$x = \frac{10k}{3} \qquad x = \frac{10}{7}$

$\frac{10}{3}k = \frac{10}{7}$

$k = \frac{3}{7}$

20 $CDEF$ is a quadrilateral.



$\vec{CD} = \mathbf{a}$, $\vec{DE} = \mathbf{b}$ and $\vec{FC} = \mathbf{a} - \mathbf{b}$.

(a) Express $\vec{FE}$ in terms of $\mathbf{a}$ and/or $\mathbf{b}$.
Give your answer in its simplest form.

$a - b + a + b$

$\vec{FE} = \underline{2a}$

M is the midpoint of DE.
X is the point on FM such that $FX : XM = n : 1$
CXE is a straight line.

(b) Work out the value of n.

$\vec{FX} = n\left(2a - \frac{1}{2}b\right)$

$= 2an - \frac{1}{2}bn$

$\vec{CX} = \vec{CF} + \vec{FX}$

$= -a + b + 2an - \frac{1}{2}bn$

$= \underbrace{2an - a} + \underbrace{b - \frac{1}{2}bn}$

$\vec{FM} = a - b + a + \frac{1}{2}b$

$= 2a - \frac{1}{2}b$

$\vec{CE} = \underline{a + b}$

$\vec{CX} = a\underbrace{(2n-1)} + b\underbrace{\left(1 - \frac{1}{2}n\right)}$

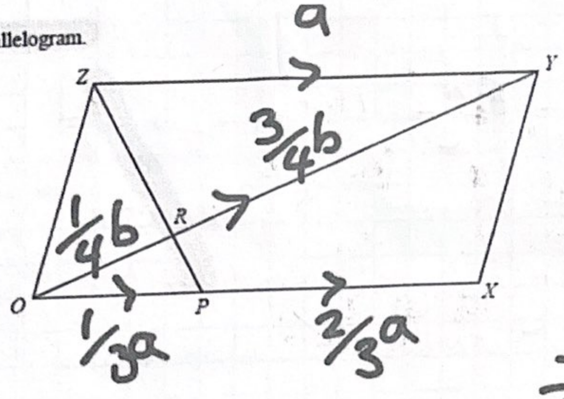
$2n - 1 = 1 - \frac{1}{2}n \qquad 2\frac{1}{2}n = 2$

$2\frac{1}{2}n - 1 = 1 \longrightarrow n = 2 \times \frac{2}{5} = \frac{4}{5}$

$\frac{4}{5} : \frac{1}{5}$

$4:1$

24 $OXYZ$ is a parallelogram.



$\overrightarrow{OX} = \mathbf{a}$
$\overrightarrow{OY} = \mathbf{b}$

P is the point on OX such that $OP : PX = 1 : 2$
R is the point on OY such that $OR : RY = 1 : 3$

Work out, in its simplest form, the ratio $ZP : ZR$
You must show all your working.

$$\overrightarrow{ZP} = a - b + \frac{1}{3}a$$
$$= \frac{4}{3}a - b$$
$$= \frac{1}{3}(4a - 3b)$$

$$\overrightarrow{ZR} = a - \frac{3}{4}b$$
$$= \frac{1}{4}(4a - 3b)$$

$ZP : ZR$

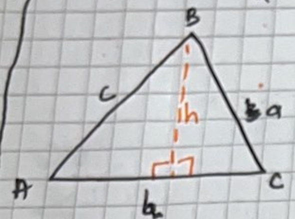
$\times 4 \quad \frac{1}{3} : \frac{1}{4} \quad \times 3$

$\frac{4}{12} : \frac{3}{12}$

$4 : 3$

Trigonometry #2

Proof of $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$



① $\sin A = \frac{opp}{hypotenuse} = \frac{h}{c}$

$\sin A = \frac{h}{c}$

$c \sin A = h$

② $\sin C = \frac{opp}{hyp} = \frac{h}{a}$

$\sin C = \frac{h}{a}$

$a \sin C = h$

③ therefore

$c \sin A = a \sin C$

$\frac{\sin A}{a} = \frac{\sin C}{c}$

note
Toprove Sin B
- relabel
- proceed with same argument

$a^2 = b^2 + c^2 - 2bc\cos A$

2 lengths → $b^2 + c^2$

Angle in between → $2bc\cos A$

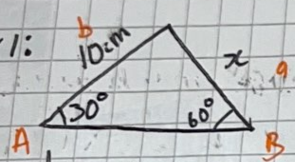
Cosine Rule

There are 2 versions:

$a^2 = b^2 + c^2 - 2bc\cos A$ (angles)

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ (sides)

EX1:



Find x

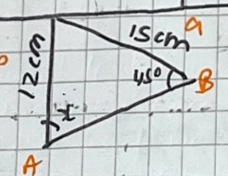
$\frac{\sin(30°)}{x} = \frac{\sin(60°)}{10}$

$\frac{x}{\sin(30)} = \frac{10}{\sin(60)}$

$x = \frac{10}{\sin(60°)} \times \sin(30°)$

$x = 5.77\,cm$

EX2:



$\frac{\sin(x)}{15} = \frac{\sin(45)}{12}$

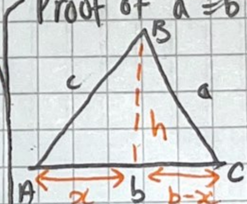
$\cancel{\frac{15}{\sin(x)} = \frac{12}{\sin}}$

$\sin(x) = \frac{\sin(45)}{12} \times 15$

$\sin^{-1}\left(\frac{\sin(45)}{12} \times 15\right) = x$

$x = 62.1°$

Proof of $a^2 = b^2 + c^2 - 2bc\cos A$



① $\cos A = \frac{adj}{hyp} = \frac{x}{c}$

$c \times \cos A = x$

②

$h^2 + x^2 = c^2$

$h^2 + (b-x)^2 = a^2$

↓ expands to

$h^2 + b^2 - 2bx + x^2 = a^2$

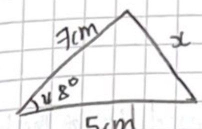
Substitute in

$h^2 + b^2 - 2bc\cos A + x^2 = a^2$

Substitute in

$b^2 - 2bc\cos A + c^2 = a^2$

EX3:



Find x.

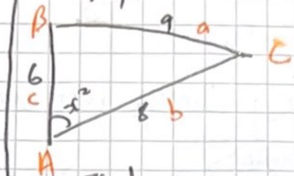
$a^2 = b^2 + c^2 - 2bc\cos A$

$x^2 = 7^2 + 5^2 - (2 \times 7 \times 5 \times \cos(48))$

$x^2 = 27.16$

$x = 5.21cm$

EX4:



Find x

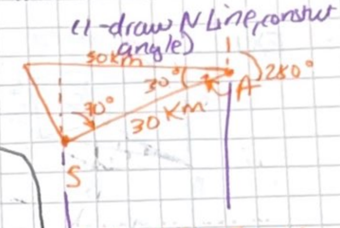
$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\cos x = \frac{8^2 + 6^2 - 9^2}{2 \times 6 \times 8} = \frac{19}{96}$

$\cos^{-1}\left(\frac{19}{96}\right) = 78.6°$

EX5: I travel 30Km on a bearing of 70°. Then travel 50Km on a bearing of 280°. How far am I from where I started?

(1-draw N Line, construct angle)



Alternate Z angles

$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 30^2 + 50^2 - (2 \times 30 \times 50 \times \cos 30°)$

$a^2 = 801.92$

$a = 28.3Km$

WHEN TO CHOOSE:

SINE RULE

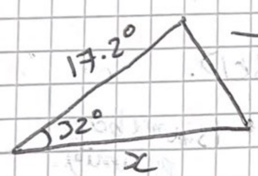
- Need 2 angles
  opposite pairs (angle, opposite side)

COSINE RULE

- Need 3 sides /
  2 ~~angles~~ sides and angle in between
  ↓
  included angle

- Using sine rule and cosine rule to find the area of a triangle

EX6:



The Area of this triangle is $40cm^2$.
Find x.

$$\boxed{Area = \frac{1}{2}ab\sin C}$$

$40 = \frac{1}{2} \times 17.2 \times x \times \sin(32°)$

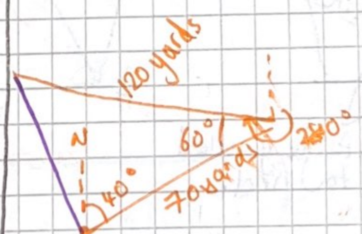
$40 = \frac{17.2x}{2} \times \sin(32°)$

$80 \div 17.2x = \sin(32)$

$x = \sin(32) \times \frac{80}{17.2}$

$x = 8.78$

EX7: I walk 70 yards on a bearing of 40°. I then walk 120 yards on a bearing of 280°. How far am I from where I started?



with all the angles I can find the 3rd length

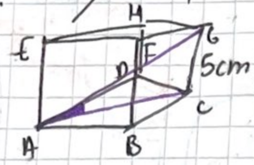
$a^2 = 120^2 + 70^2 - 2 \times 120 \times 70 \times \cos 60°$

$a^2 = 10900$

$a = 104 yards$

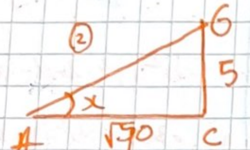
3 Dimensional trig

EX1: Cuboid

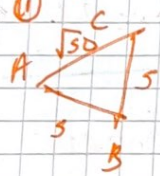


Find $G\hat{A}C$.

②



①



③

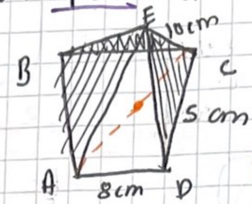
$\tan(x) = \frac{5}{\sqrt{50}}$

$\tan^{-1}\left(\frac{5}{\sqrt{50}}\right) = x$

$x = 35.26$

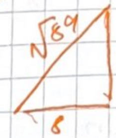
NOTE:
Find the exact value
means surd / fraction

EX2: The Apex (top point) is directly over the middle of the base.



a.) Calculate the height

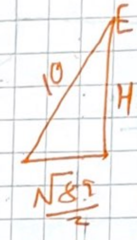
① Figure out $\vec{AC}$, then $\div 2$



$\boxed{8^2 + 5^2 = 89}$

$A \quad \frac{\sqrt{89}}{2}$

②

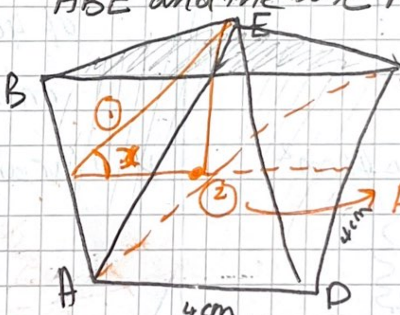


$(10^2) - \left(\frac{\sqrt{89}}{2}\right)^2 = H$

$H^2 = 77.75$

$H = 8.82cm$

b.) Calculate the angle between a face ABE and the base ABCD.

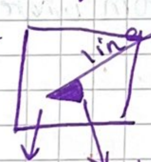


(square based pyramid)

② Half length of the base

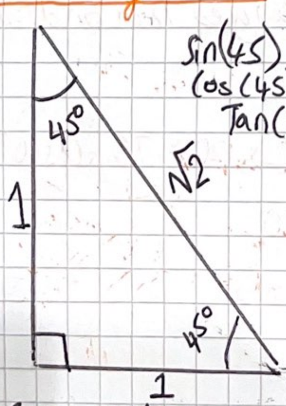
(NOT very good example)

for these questions, you get



Plane Angle you need to find!

Exact Trig Ratios

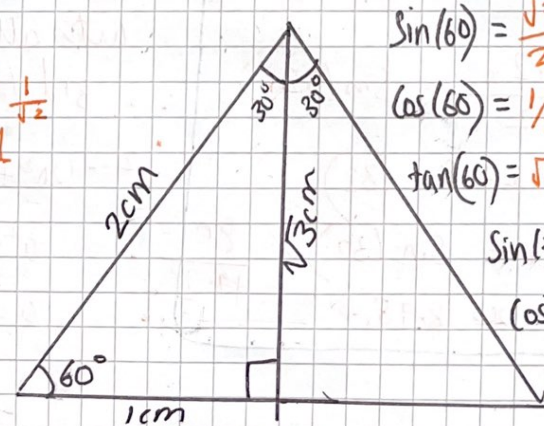


$\sin(45) = \frac{1}{\sqrt{2}}$

$\cos(45) = \frac{1}{\sqrt{2}}$

$\tan(45) = 1$

Right Angle Isosceles



Equilateral Triangle

$\sin(60) = \frac{\sqrt{3}}{2}$

$\cos(60) = \frac{1}{2}$

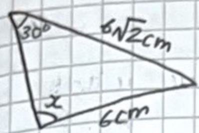
$\tan(60) = \sqrt{3}$

$\sin(30) = \frac{1}{2}$

$\cos(30) = \frac{\sqrt{3}}{2}$

$\tan(30) = \frac{1}{\sqrt{3}}$

EX1: find x (non calc)



① Use Sine Rule

$$\frac{\sin(30°)}{6} = \frac{\sin(x)}{6\sqrt{2}}$$

② can x both by 6

$$\sin(30) = \frac{\sin(x)}{\sqrt{2}}$$

③ memorised value of sin(30) is...

$$\frac{1}{2} = \frac{\sin(x)}{\sqrt{2}}$$

$$\frac{\sqrt{2}}{2} = \sin(x)$$

change surd to be denominator

$$\frac{\cancel{\sqrt{2}}}{2} \cdot \frac{1}{\sqrt{2}} = \sin(x)$$

so $\sin(x) = \frac{1}{\sqrt{2}}$

(memorised) $x = 45°$

but there's more than one answer...

EX2: $\sin(x) = \frac{\sqrt{3}}{2}$ for

$0 \leq x \leq 360$

Find the value(s) of x.

Use rules

memorised value $\sin(60) = \frac{\sqrt{3}}{2}$

$180 - 60 = 120$

So $x = 60°, 120°$

Key Rules

To find both values of...

Sin 180 − CALC

Cos 360 − CALC

Tan 180 + CALC

Thus... ± 360 (sin or cos)

± 180 (tan)

EX3: If $\sin(45) = \frac{1}{\sqrt{2}}$, what is

$\sin(135)$ Add to 180° so $\frac{1}{\sqrt{2}}$

$\sin(225)$ is $-\frac{1}{\sqrt{2}}$

$\sin(-315)$ $\frac{1}{\sqrt{2}}$

If $\cos(60) = 1/2$ what is

$\cos(120)$ $-1/2$

$\cos(300)$ $1/2$

$\cos(-300)$ $1/2$

If $\tan(30°) = \frac{1}{\sqrt{3}}$

$\tan(210°) = -1/\sqrt{3}$

$\tan(-150°) = -1/\sqrt{3}$

$\tan(120°) = \frac{1}{\sqrt{3}}$

How to know when to change the signs:

$\sin(180+x) = -\sin(x)$

$\sin(180-x) = \sin(x)$

$\sin(-x) = -\sin(x)$

$\cos(x) = \cos(-x)$

$\cos(x) = \cos(360-x)$

$\cos(x) = \cos(360+x)$

$\cos(x) = -\cos(180-x)$

$\tan(x) = \tan(180+x)$

$-\tan(x) = \tan(-x)$

$-\tan(x) = \tan(90+x)$

Solving trig equations

EX1: Solve $\sin x = 4/5$ for $0 \leq x \leq 360$

(2 answers because it hits x2 over 360)

$x = \sin^{-1}(4/5)$

$x = 53.1°$ [1st ANS]

180° − Calc

$x = 126.9°$ [2nd ANS]

EX2: Solve $2\cos x + 1 = 0$ for $0 \leq x \leq 360$

$2\cos x = -1$

$\cos x = -\frac{1}{2}$

[1st ANS] $x = 120°$

[2nd ANS] $360 - 120$ $x = 240°$

EX3: Solve $3\sin^2 x = \frac{1}{3}$ for $0 \leq x \leq 360$

$\sin^2 x = \frac{1}{9}$

$\sin x = +\frac{1}{3}$	$\sin x = -\frac{1}{3}$
$x = 19.5°$	$x = -19.5°$ not in range
or	or (180−x)
$x = 160.5°$	$x = 199.5°$
	(+360)
	$x = 340.5°$

Graph transformations

RULES:

$y = f(x) + K \longrightarrow$ translate by vector $\binom{0}{K}$

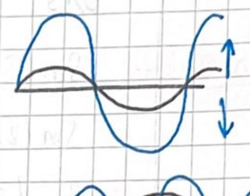
$y = f(x - K) \longrightarrow$ translate by vector $\binom{K}{0}$

$y = f(-x) \longrightarrow$ Reflect in y-axis   EG: $f(x) = x^2 - 4x \rightarrow f(-x) = (-x)^2 - 4(-x)$ $= x^2 + 4x$

$y = -f(x) \longrightarrow$ Reflect in x-axis   EG: $f(x) = x^2 - 4x$ $-f(x) = -1(x^2 - 4x)$ $= -x^2 + 4x$

Further maths

$y = Kf(x) \longrightarrow$ Stretch in y direction Scale factor x



$y = f(Kx) \longrightarrow$ Stretch in x-direction Scale factor $1/K$ (squash!)

Algebraic fractions and Proof

Simplifying Algebraic fractions

EX1:1 $\frac{AX^2}{A^2X} = \frac{X}{A}$

1:2 $\frac{x+1}{x}$ CANNOT SIMPLIFY!

1:3 $\frac{x^2+x}{x^2-1}$ factorise to cancel down → diff of 2 squares $\frac{x(x+1)}{(x+1)(x-1)} = \frac{x}{x-1}$

1:4 $\frac{2y^2+3y+1}{y^2+8y+7}$ $\frac{2y^2+2y+1y+1 \rightarrow 2y(y+1)+1(y+1)}{(y+7)(y+1)}$ $\frac{(2y+1)\cancel{(y+1)}}{(y+7)\cancel{(y+1)}}$ $\Leftarrow$ $\frac{(2y+1)}{(y+7)}$

1:5 $\frac{3}{x} + \frac{4}{5x}$ ×5

$\frac{15}{5x} + \frac{4}{5x} = \frac{19}{5x}$

1:6 $\frac{1}{x-2} + \frac{7x}{x+3}$ (cross multiply) $\frac{x+3+7x(x-2)}{(x-2)(x+3)}$

$= \frac{7x^2-13x+3}{(x-2)(x+3)}$

1:7 $\frac{6}{a-b} - \frac{2}{b-a}$

since $a-b = -(b-a)$

$\frac{6}{a-b} - \left(-\frac{2}{b-a}\right)$

$\frac{6}{a-b} + \frac{2}{a-b} = \frac{8}{a-b}$

1:8 $\frac{6x^{\cancel{3}2}y^2}{\cancel{5}a\cancel{b^2}} \times \frac{10a^{\cancel{3}2}\cancel{b}}{2x}$

$\frac{6x^2y^2a^2}{b}$

1:9 $\frac{2n^2-5n-3}{2n^2+5n+2} \div \frac{n^2-9}{n^2-4}$ Flip and factorise

$\frac{\cancel{(2n+1)}\cancel{(n-3)}}{\cancel{(2n+1)}\cancel{(n+2)}} \times \frac{(n-2)\cancel{(n+2)}}{\cancel{(n-3)}(n+3)}$

$\frac{n-2}{n+3}$

Mr Nicholls travelled 300Km to see his girlfriend. On his return journey home, his average speed was increased by 20Km per hour and the time of his journey decreased by 1hour and 15 minutes.
Find his average speed on the outward journey.

- let V be average speed on outward
- let T be time taken on outward journey

- Average speed = $\frac{\text{total distance}}{\text{total time}}$

- outward journey: • Return journey

(a) $V = \frac{300}{t}$ (b) $V + 20 = \frac{300}{t - 1.25}$

- Now we have simultaneous equations, plug (a) in (b)

$$\frac{300}{t} + 20 = \frac{300}{t - 1.25}$$

tip: Multiply by both denominators here: $\times\, t\,(t - 1.25)$

$$300(t - 1.25) + 20t(t - 1.25) = 300t$$

$$300t - 375 + 20t^2 - 25t = 300t$$

$$20t^2 - 25t - 375 = 0$$

$\div 5$

$$4t^2 - 5t - 75 = 0$$

$$(4t + 15)(T - 5) = 0$$

$$T = 5$$

$$V = \frac{300}{5} = 60\text{Km/h}$$

EX1: Prove that an odd number plus an odd number is always even.

Let $m = 2p + 1$
$n = 2q + 1$

Where 'p' and 'q' are natural numbers

(AKA ↑ $p, q \in \mathbb{Z}^+$ — Positive integers; $\in$ are members of)

$$2p + 1 + 2q + 1$$
$$= 2p + 2q + 2$$
$$= 2(p + q + 1)$$

This is clearly a multiple of 2 and therefore even.

EX2: Prove that the difference between the squares of 2 consecutive numbers is odd

Let n be a whole number

then $(n+1)$ is the consecutive whole number.

Then $(n+1)^2 - n^2$ (squared difference)

$$n^2 + 2n + 1 - n^2$$
$$= 2n + 1$$

This is clearly odd as it is one more than a multiple of 2.

QED (add this to conclude a proof)

EX3: Prove that $9^n - 1$ is a multiple of 8 (always)

- $9^n = (3^2)^n$
- $9^n - 1 = (3^2)^n - 1$
- $= 3^{2n} - 1$
- Difference of 2 squares
- $(3^n - 1)(3^n + 1)$
- $3^n - 1$ must be even as it is one less than an odd number

- Similarly $3^n + 1$ must be even as it is one more than an odd number.
- As they are consecutive even numbers one must be a multiple of 4, and the other a multiple of 2. → (2 4 6 8 10 12)

Hence, a multiple of 4 multiplied by a multiple of 2 is a multiple of 8.

Capture, Recapture:

EX1: Thomas wants to estimate the number of rabbits in a field. She captures 24 rabbits, puts a mark on them, and releases them back into the field. The next day she returns to the field and catches 50 rabbits - 4 of these have a mark on them. Estimate the total number of rabbits in the field.

KEY POINT: Proportion of rabbits tagged in population = Proportion of rabbits tagged in sample

- Proportion with a tag = $\frac{4}{50}$
- (so by the key point above, we then expect $4/50$ of the whole rabbit population to be tagged.)

• $\frac{4}{50}$ of Rabbits = 24 rabbits → because Thomas tagged them

↓

To find total Population we want $\frac{50}{50}$ of Rabbits so find $\frac{1}{50}$ and get $\frac{50}{50}$:

$\frac{1}{50} = \frac{24}{4} = 6$ so $\frac{50}{50}$ $(\times 50)$ $= 6 \times 50$

$= \underline{300 \text{ rabbits}}$

(b) What assumptions are you making?

- We are assuming that:
  - none of the rabbits have died
  - none of the rabbits have lost their tag
  - the proportion of the rabbits sampled reflect the proportion of the rabbits tagged in the whole field.

CALCULATOR TRICKS

- Product of Prime factors shift + FACT
- graphs: menu, option 9 (table),

  f(x) = ______
  type in equation

  x = ALPHA + X,

  skip g(x)

  start = ______ end = ______
  Ex between -3 and 1

  Get table!

- solving simultaneous equations with 2 or 3 uknowns

menu, option A, simultaneous equations select

- Quadratic Inequalities

  menu, option B, polynomial degree = 2,
  then get answers!

- Quadratic equations

  if you press = again works out turning point!

menu, A, option 2 polynomial, degree = highest Power (2)

• Factorising Quadratics

Menu, A, 2, degree = 2, getsolutions so

: 4 and -3 is $(x-4)(x+3)$

• Mean of freq table

Menu, 6, pick 1-variable, enter, midpoints! in x, Press $\boxed{AC}$

OPT, Press 1-var calculation

$\bar{x}$ (mean)

• Mixed + improper fractions

SHIFT + SD

• Units

Shift + 8